

Further Results on Online Node- and Edge Deletion Problems with Advice

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Online Algorithms: A Short Recap

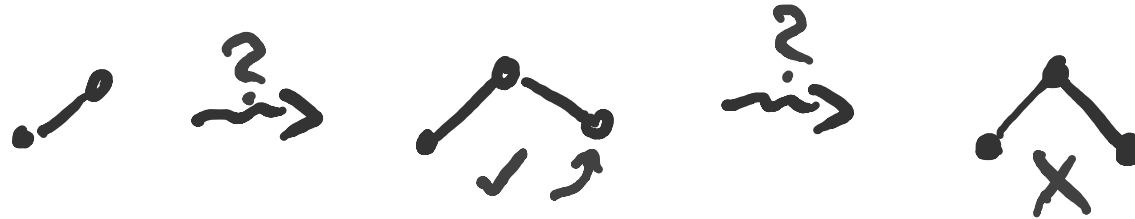
Input iterative:



Immediate decisions:



|Input| unknown:



Performance measure:
(p_{ALG} target fct)

$$\frac{p_{\text{ONLINE}}}{p_{\text{BEST}}} := \text{Competitive Ratio}$$

Example: Vertex Cover

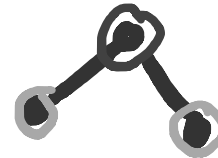
1.



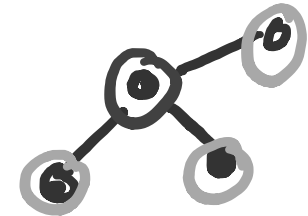
2.



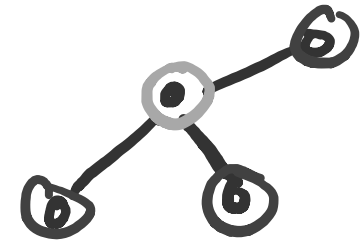
3.



4.



Test:



1.



2.



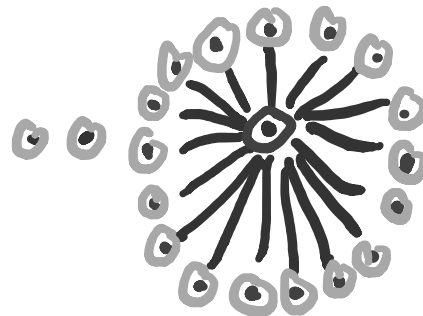
3.



4.



$n \rightarrow \infty$



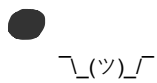
$$\leq \frac{3}{2}$$

Why decide anything in steps 1-3?

Delayed Decisions

Only act if necessary!

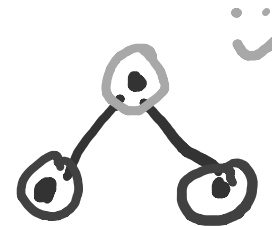
1.



2.



3.



Allowed to take Elements into solution later

Delayed Decisions: Vertex Cover

Simple Algo:



(a.k.a. 2-Approximation
for VC)

Simple Lower bound:

1.



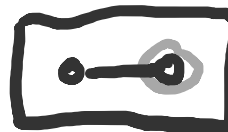
2.



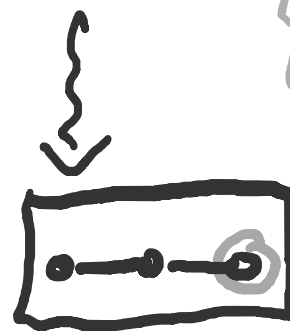
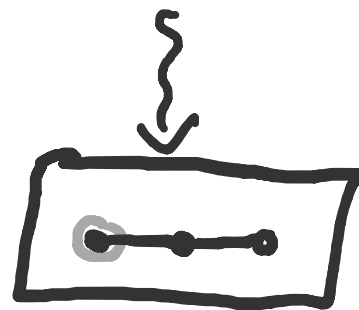
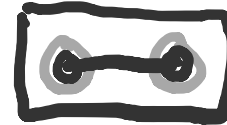
2.1



2.2



2.3



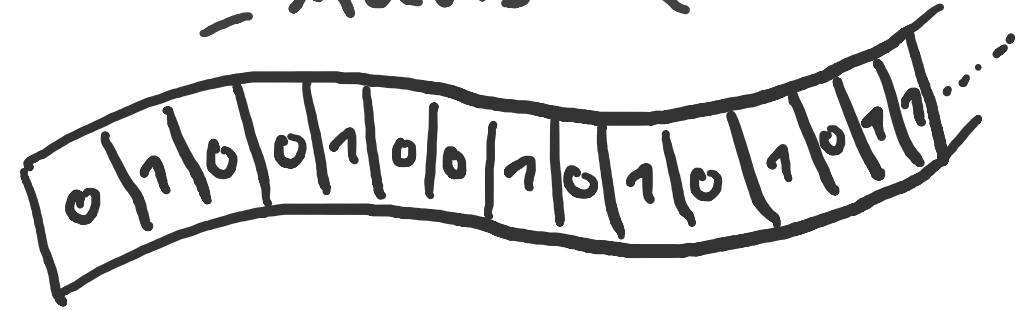
strictly 2-competitive!

Advice Complexity

Advisor

Online Algorithm + Online Problem

+



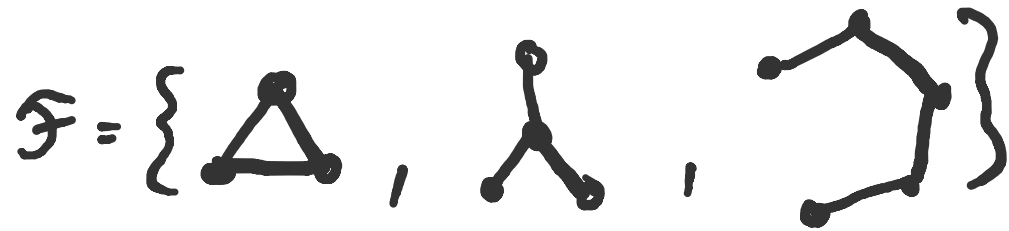
sometimes not competitive ☹️

Optimal / Good competitive ratio! 😊

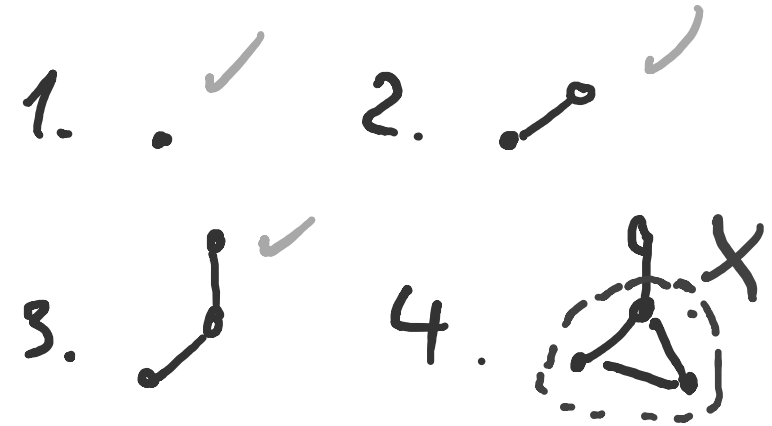
How much advice necessary / sufficient?

Online Node- and Edge Deletion

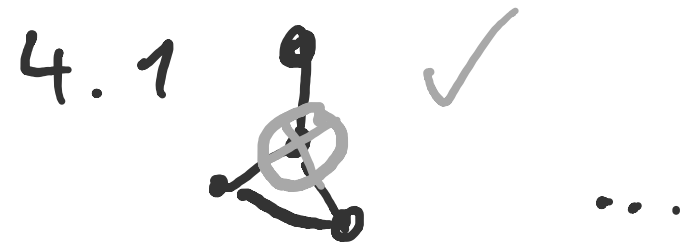
Family of graphs:



Should not be induced



Node- or Edge-deletion
necessary!



An Overview over Problem Types

Distinctions: - Delete Nodes or Edges?



- Single graph forbidden or family?

$$\mathcal{F} = \{\lambda\} \text{ or } \mathcal{F} = \{\Delta, \square, \lambda, \dots\}$$

- Graphs connected?

$$\mathcal{F} = \{\Delta, \square, \lambda, \dots\} \text{ or } \mathcal{F} = \{\lambda, \Delta \square, \dots\}$$

The Full Tables

Online <u>Node</u> Deletion	Single Graph H forbidden	Family \mathcal{F} forbidden
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All Graphs connected	Ross 18	Ross 18
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Arbitrary Graphs	This work	?
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Online <u>Edge</u> Deletion	Single Graph H forbidden	Family \mathcal{F} forbidden
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All Graphs connected	Ross 18	This work
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Arbitrary Graphs	?	?
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H-Node-Deletion: Sketch LB

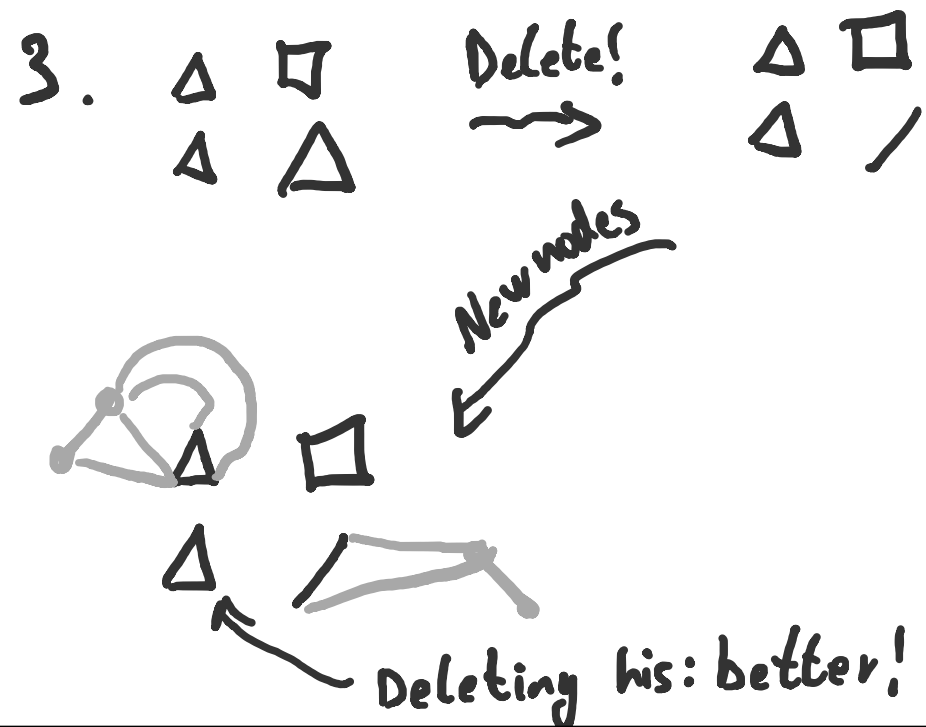
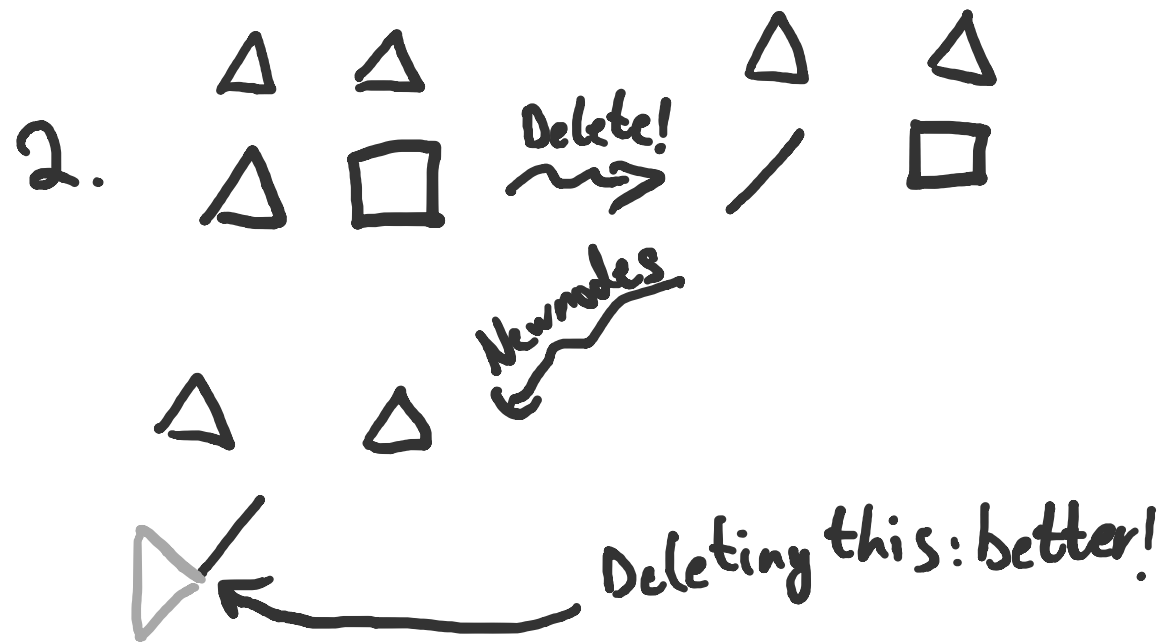
$H = \triangle \diamond \triangle \triangle$

Enough to delete all \square
or all but 2 \triangle !

What can an adversary do?

1. $G = \diamond \begin{matrix} \triangle \triangle \triangle \triangle \triangle \\ \triangle \triangle \triangle \triangle \triangle \\ \triangle \dots \end{matrix}$

Problem if we delete only \triangle !



Results for H-Node-Deletion

Lower bound:

Theorem 1. *Let H be a graph. Let C_{max} be a component of H of maximum size. Any online algorithm optimally solving the DELAYED H -NODE DELETION PROBLEM uses at least $opt_H(G) \cdot \log |V(C_{max})| + (\nu_{C_{max}}(H) - 1) \cdot \log(opt_H(G))$ many advice bits on input G .*

Upper bound:
(not in this talk)

Theorem 2. *Let H be a graph. Let $C_{min\nu} = \arg \min_{C \in C_H} \{\nu_C(G) - \nu_C(H)\}$ and $C_{min} = \arg \min_{C \in C_{min\nu}} \{|V(C)|\}$. The DELAYED H -NODE DELETION PROBLEM can be solved optimally using at most $opt_H(G) \cdot \log |V(C_{min})| + O(\log opt_H(G))$ many advice bits on input G .*

Deleting Edges from Connected Graphs

Upper bound: $\mathcal{F} = \{\Delta, \square\}$

forbidden



Advice:

		3				
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calculate „good“ sets

$\text{good}_{\Delta} = \{ \nearrow, \searrow, \dashrightarrow, \nwarrow, \dots \}$

$\text{good}_{\square} = \{ \dots \}$

Index the sets!

Advice $\hat{=}$ Index

Advice complexity: A bit involved

Theorem 3. Let $\mathcal{F} = \{H_1, \dots, H_k\}$ be a set of connected graphs and let S_i be H_i -sound and H_i -sufficient for all $i \in \{1, \dots, k\}$. Then there is an $m \in \mathbf{R}$ and an algorithm that solves the DELAYED CONNECTED \mathcal{F} -EDGE DELETION PROBLEM for every graph G with $m \cdot \text{opt}_{\mathcal{F}}(G) + O(1)$ many advice bits where $2^m \leq \beta(S_i)$ for all $i \in \{1, \dots, k\}$.

Lower Bound: A Very Rough Overview

Fix a correct Algorithm A

Look at all sets of edges A has to distinguish w.r.t. \mathcal{F}

(These sets are computable)

Theorem 5. Let $\mathcal{F} = \{H_1, \dots, H_k\}$ be connected graphs. The advice complexity for DELAYED CONNECTED \mathcal{F} -EDGE DELETION is $m \cdot \text{opt}_{\mathcal{F}}(G) + O(1)$ where $m = \max_{i \in \{1, \dots, k\}} \min\{\log \beta(S) \mid S \subseteq 2^{E(H)}, S \text{ is } H_i\text{-sound and } H_i\text{-sufficient}\}$. There is an algorithm that can compute m from \mathcal{F} . More specifically, there is an algorithm that gets \mathcal{F} and $t \in \mathbb{N}$ as the input and returns the t th bit of the binary representation of m .

Tight bound! 😊