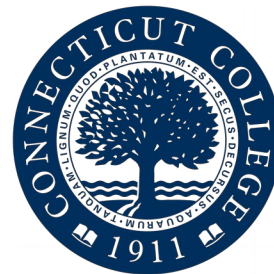


New Bounds for Maximizing Revenue in Online Dial-A-Ride

Ananya Christman¹(achristman@middlebury.edu), Christine Chung²,
Nicholas Jaczko¹, Tianzhi Li, Scott Westvold¹, Xinyue Xu¹, David Yuen

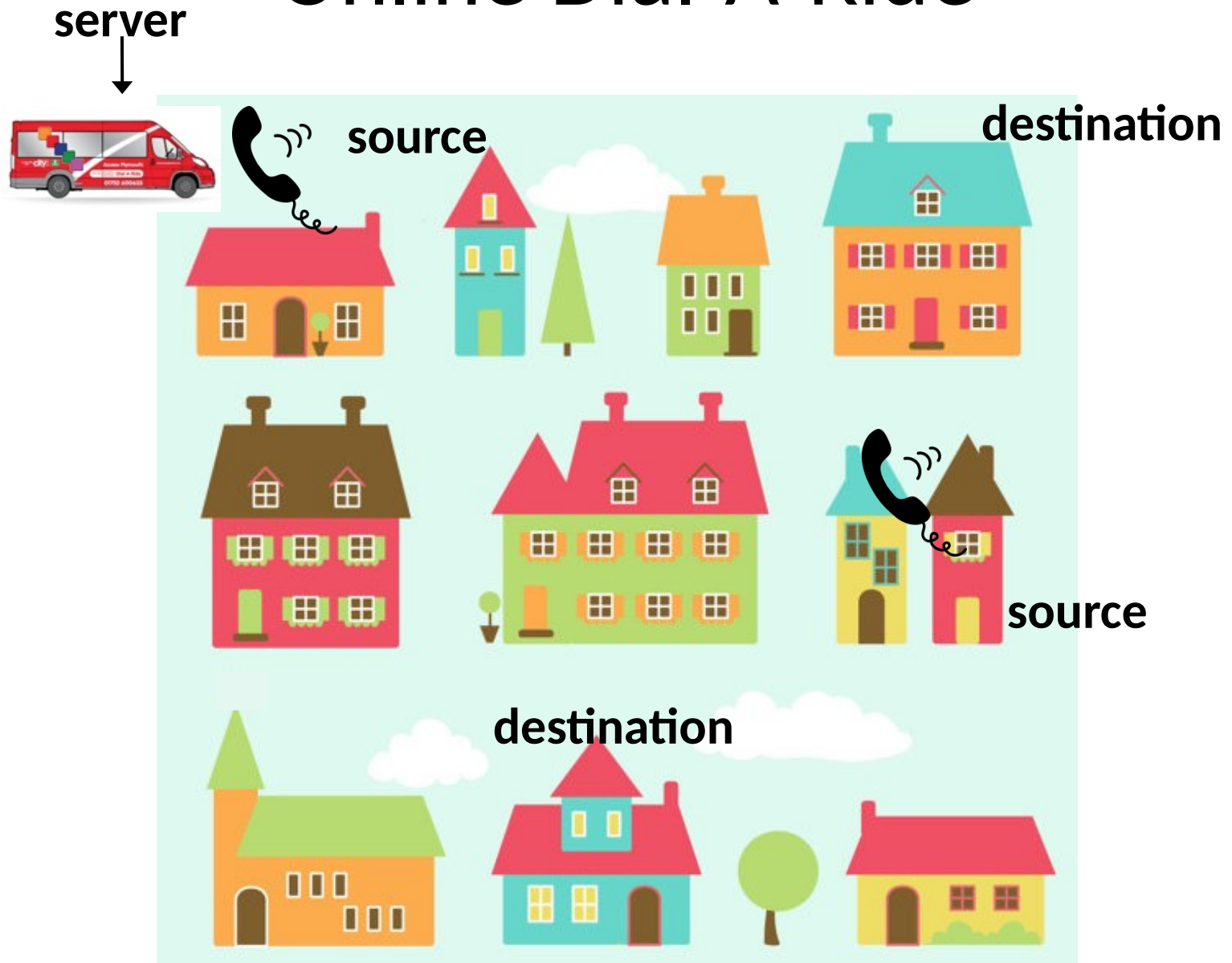


Middlebury College¹



Connecticut College²

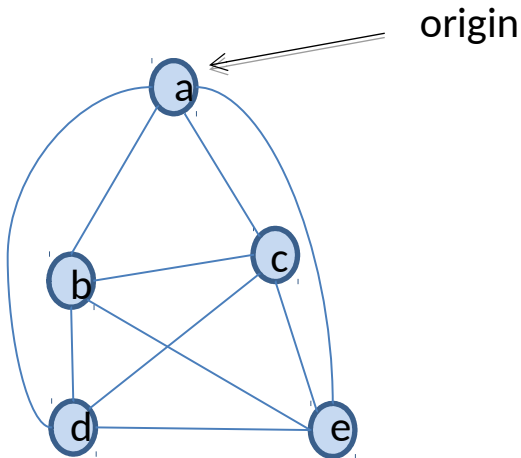
Online Dial-A-Ride



Online Dial-A-Ride

Input:

- Complete Graph (nonuniform edge weights)
- Initial Location of Server (**origin**)
- Requests: (**source**, **destination**, **release time**)



$$R1 = (a, c, 0)$$

$$R2 = (b, d, 1)$$

$$R3 = (e, a, 1)$$

$$R4 = (c, b, 3)$$

Online Dial-A-Ride with Priorities

100



Time Limit = 30

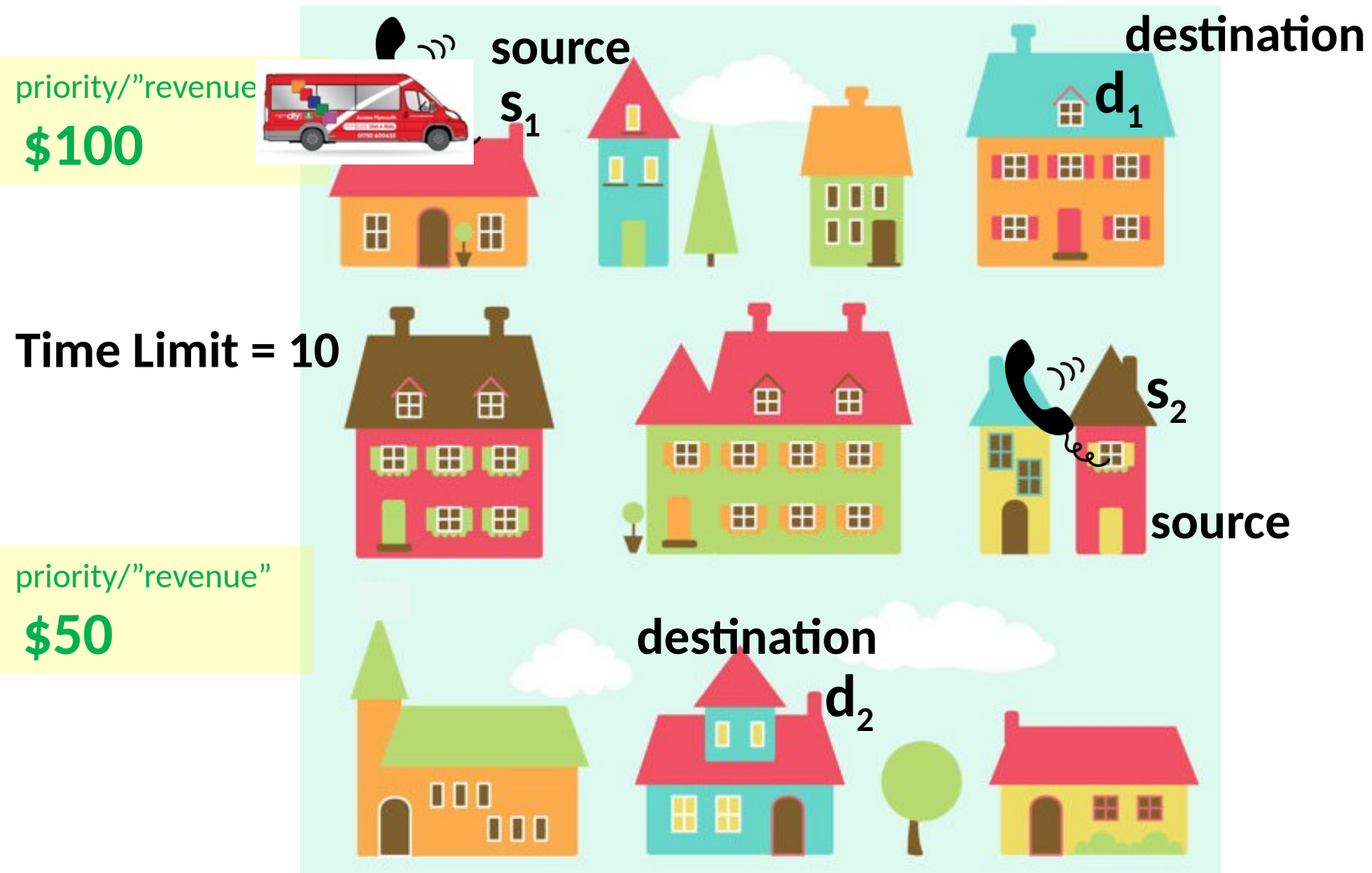


50



Revenue

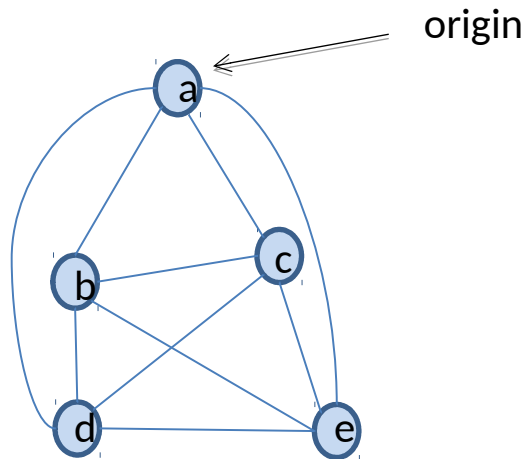
Online Dial-A-Ride with ~~Priorities~~



Online-Dial-A-Ride with Revenue (ROLDARP)

Input:

- Complete Graph (nonuniform edge times)
- Initial Location of Server (origin)
- Requests: (source, destination, release time, **priority/revenue**)
- **Time Limit (T)**



$$R1 = (a, c, 0, \$100)$$

$$R2 = (b, d, 1, \$10)$$

$$R3 = (e, a, 1, \$200)$$

$$R4 = (c, b, 3, \$50)$$

Time Limit $T = 30$

Goal:

Serve requests to **Maximize Total Revenue**

Offline version is NP-hard¹

Applications

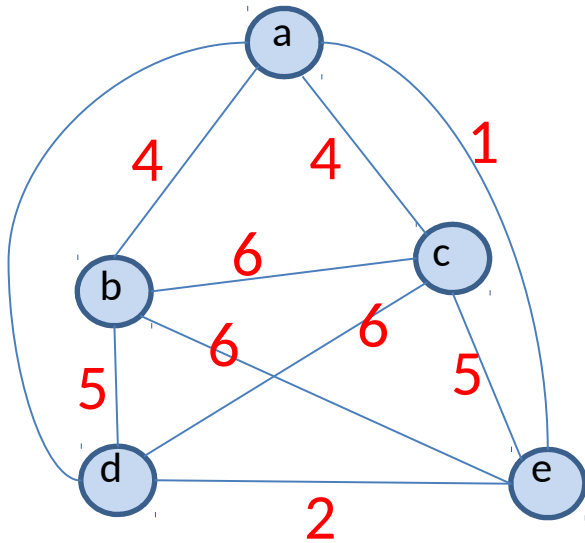


Related Work on OLDARP

- Introduced by Feuerstein and Stougie [TCS 2001] with objectives: **minimizing completion time, minimizing latency**
- **Minimizing Makespan/Minimizing Flow Time:**
 - Krumke [2002]
- **On the real line:**
 - Birx, Disser, and Schewior [APPROX/RANDOM 2019]
 - Bjelde et al. [SODA 2019]
- **Multiple servers:**
 - Ascheuer, Krumke, Rambau [STACS 2000]
- **Online Prize-Collecting Traveling Salesperson**
 - Ausiello, Bonifaci, Laura [IPL 2008]

Bounding the Edge Weights

- If edge weights may be arbitrarily large, no deterministic online algorithm exists for ROLDARP¹
- Impose a maximum edge weight T/f , for $1 < f < T$



Time Limit $T = 30$

$f = 5$

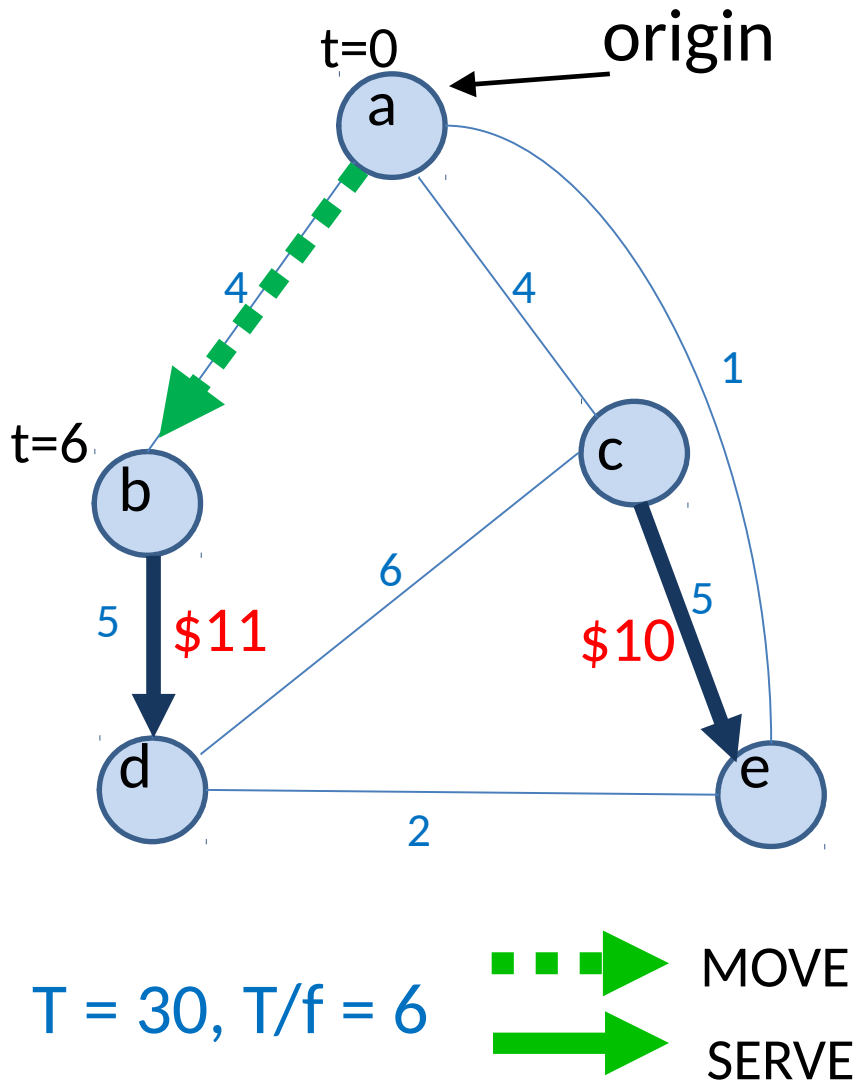
Max edge weight = $T/f = 6$

Segmented-Best-Path (SBP) Algorithm

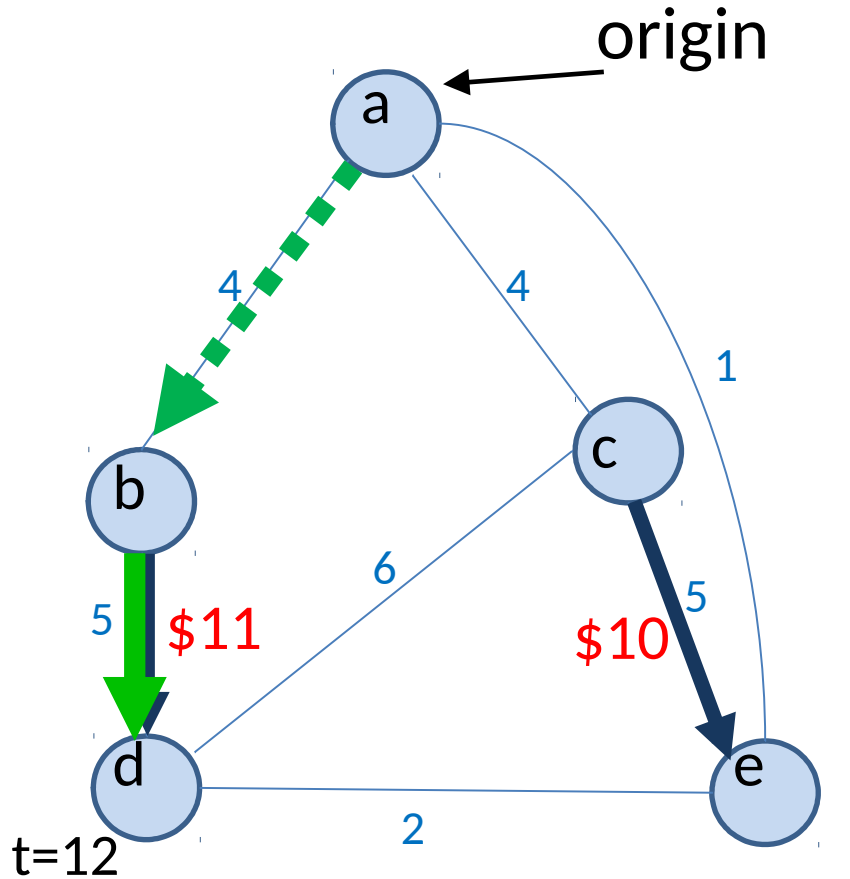
Input: G , T , $0 < f < 1$, (max edge weight is T/f)

- Split the total time T into f time “*segments*”, each of length T/f
- While there is still time remaining:
 - Alternate between:
 1. Find the *max-revenue request set* that can be served within time T/f ; let S denote this set. *Move* to the source of the first request in S .
 2. *Serve* the requests in set S .

SBP Example



SBP Example

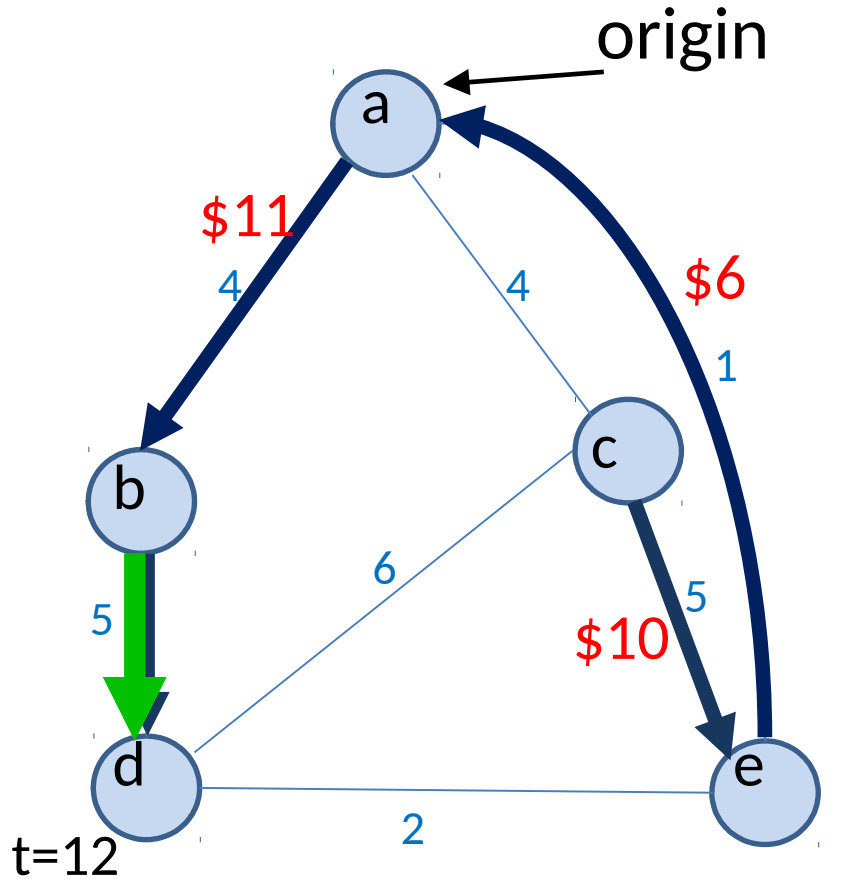


Revenue earned = \$11

$T = 30, T/f = 6$



SBP Example

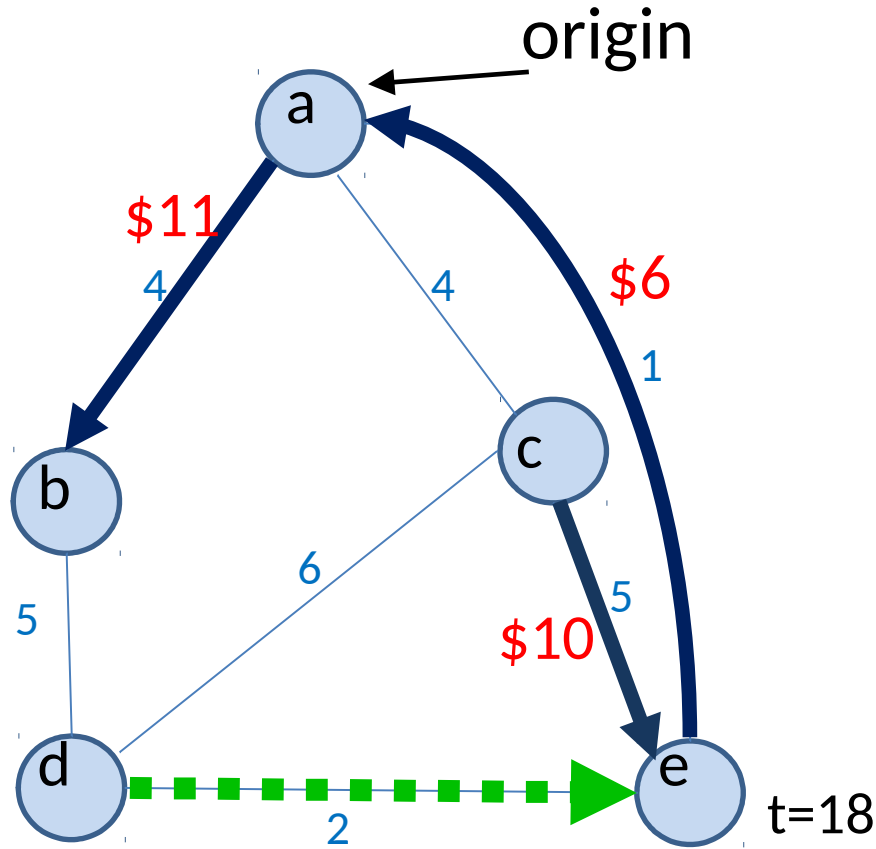


Revenue earned = \$11

$T = 30, T/f = 6$



SBP Example

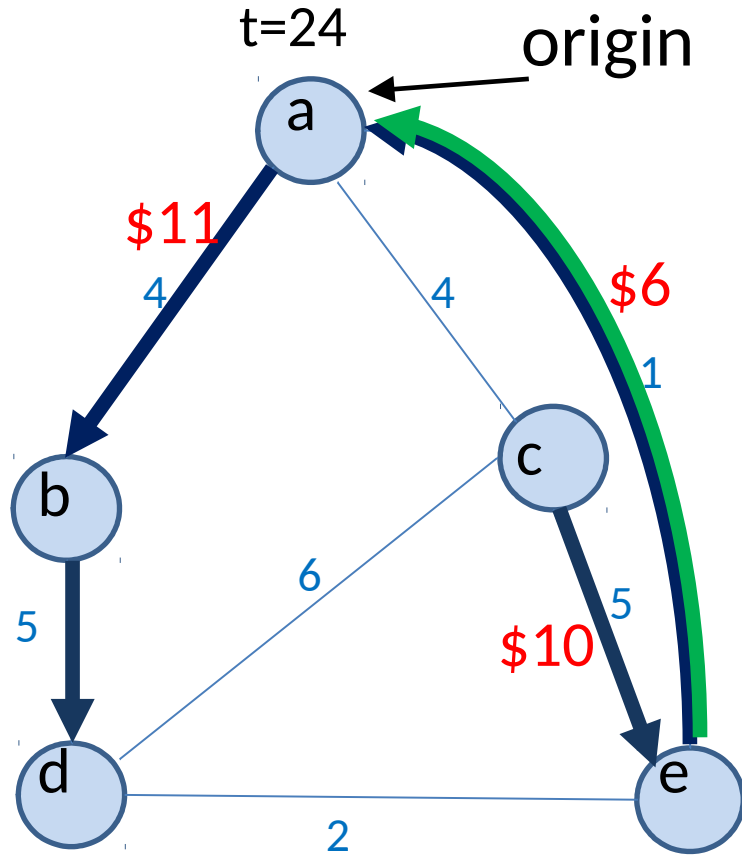


Revenue earned = \$11

$T = 30, T/f = 6$



SBP Example

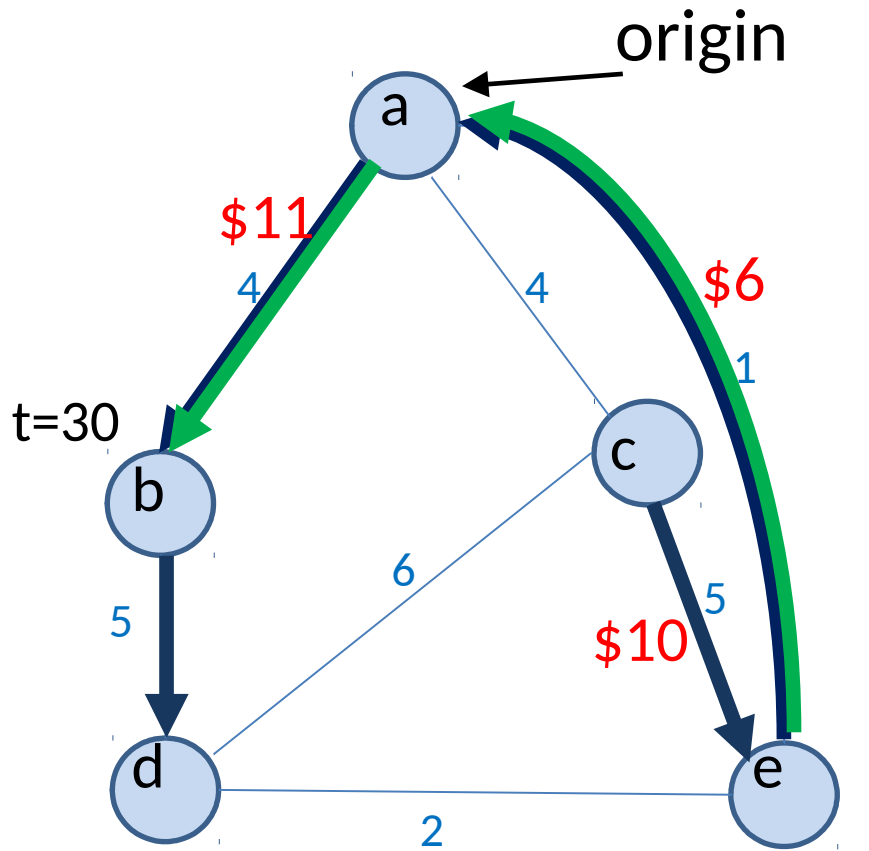


Revenue earned = $\$11 + \6

$T = 30, T/f = 6$



SBP Example



Revenue earned = $\$11 + \$6 + \$11$

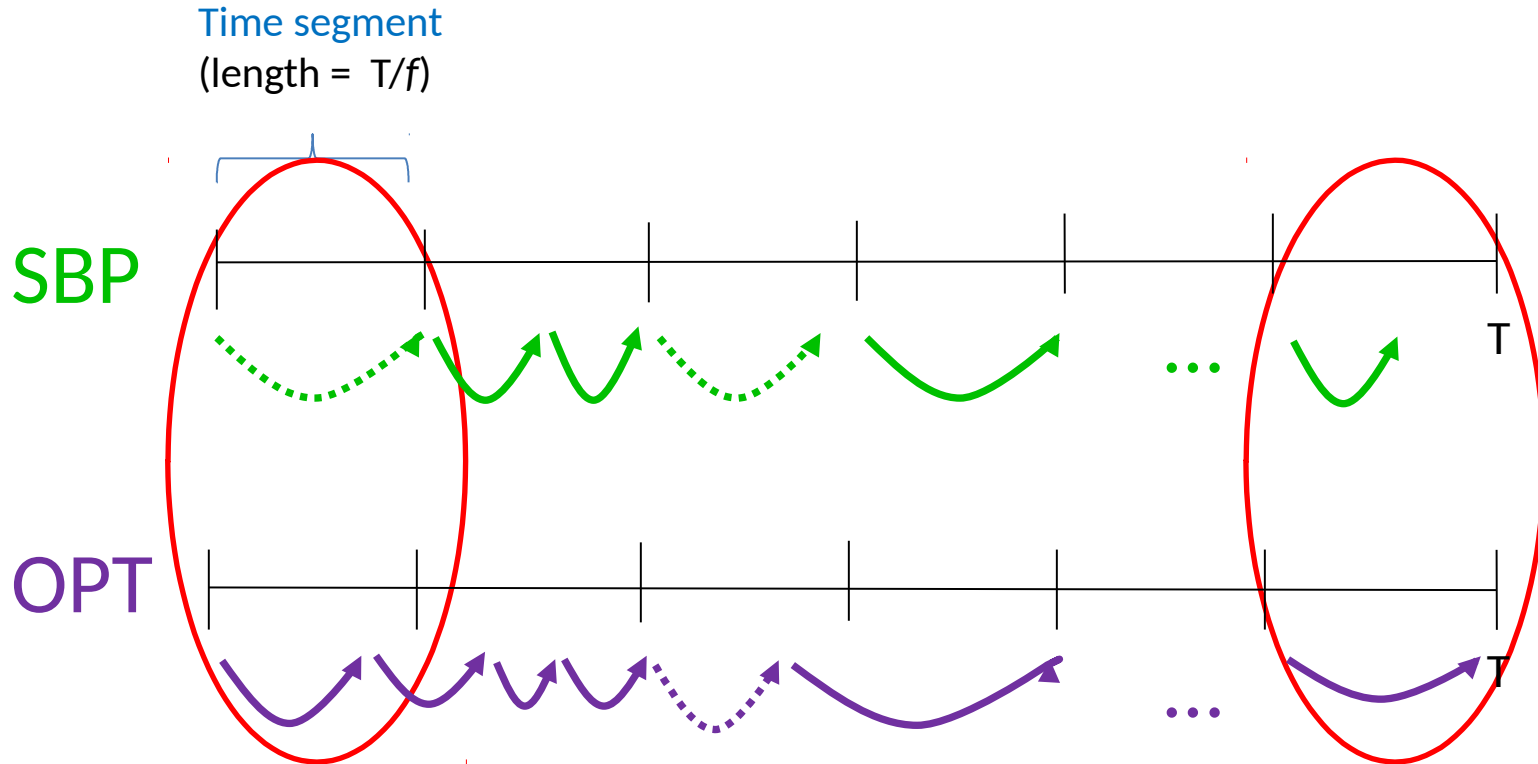
$T = 30, T/f = 6$



SBP is 5-Competitive

SBP is 5-Competitive Proof

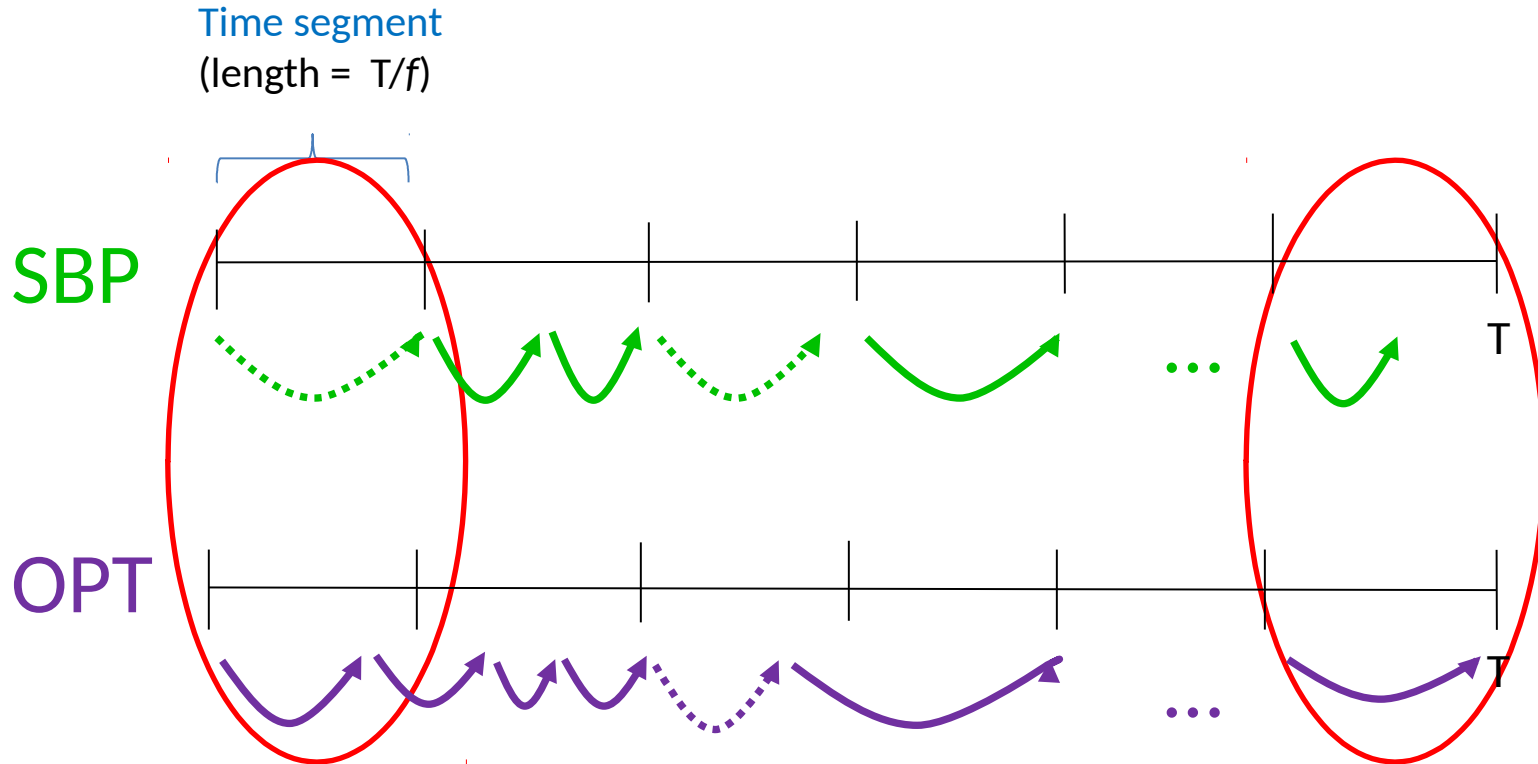
- Compare schedules of SBP and OPT, one segment at a time:



- Consider the segments where SBP earns less than OPT

SBP is 5-Competitive Proof

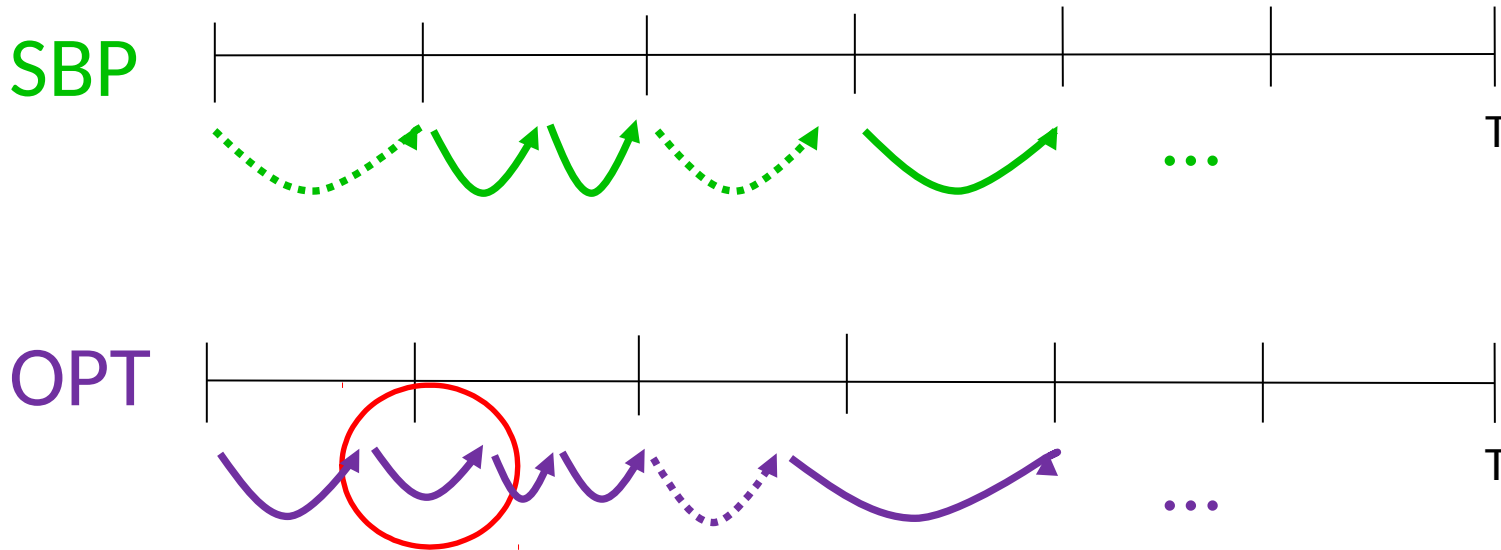
- Compare schedules of SBP and OPT, one segment at a time:



- Consider the segments where $\overline{\text{SBP}}$ earns less than $\overline{\text{OPT}}$
- Create a modified schedule $\overline{\text{SBP}}$ (initially $\overline{\text{SBP}} = \text{SBP}$) that will contain additional copies of requests
- For now: assume all requests served by OPT during a segment are available to SBP

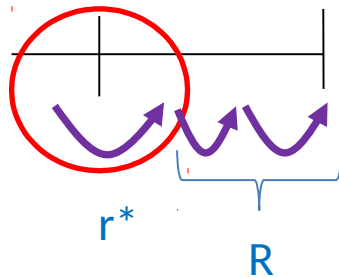
SBP is 5-Competitive Proof

- SBP does not serve any requests that **overlap** time segments



Overlapping Request

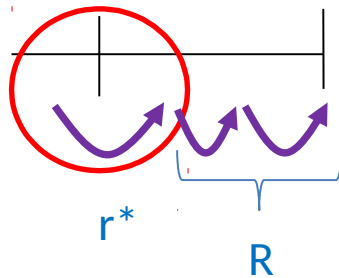
OPT



- Both r^* and R can separately be served within a time segment
- So SBP will serve the one that yields higher revenue:
 $\text{revenue}(\text{SBP}) \geq \max(r^*, R) \geq \frac{1}{2} (r^* + R) \geq \frac{1}{2} \text{revenue}(\text{OPT})$
- So SBP misses at most $\frac{1}{2}$ OPT

Overlapping Request

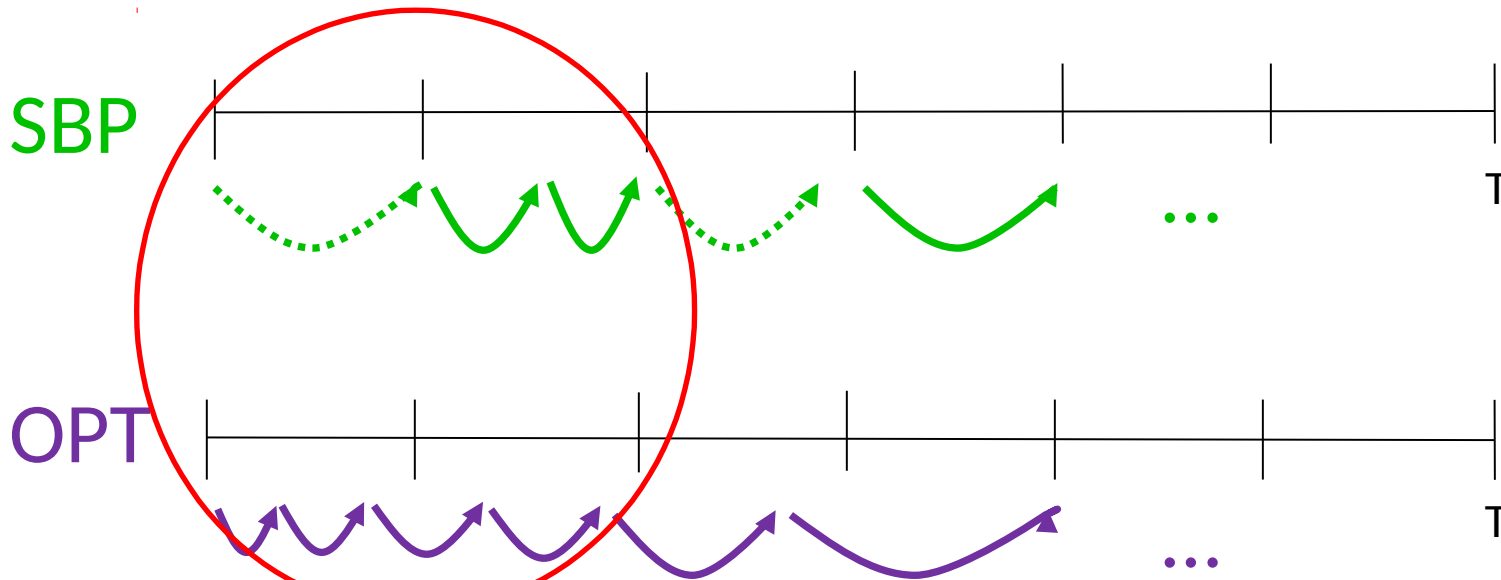
OPT



- Both r^* and R can separately be served within a time segment
- So SBP will serve the one that yields higher revenue:
 $\text{revenue}(\text{SBP}) \geq \max(r^*, R) \geq \frac{1}{2} (r^* + R) \geq \frac{1}{2} \text{revenue}(\text{OPT})$
- So SBP misses at most $\frac{1}{2}$ OPT
- So far: $\overline{\text{revenue}(\text{SBP})} \cong \text{revenue}(\text{SBP}) \bullet 2$

SBP is 5-competitive

- SBP serves in only every other time segment, OPT may serve the entire time



- For a pair of consecutive segments, SBP misses at most $\frac{1}{2}$ OPT's revenue
- So far: $\text{revenue}(\text{SBP}) \leq 2 \text{ revenue}(\text{SBP}) \cdot 2$
 $\leq 4 \text{ revenue}(\text{SBP})$

SBP is 5-competitive

- Recall: we assumed all requests served by OPT during a segment are available to SBP
- Need another factor of $\text{revenue}(\text{SBP})$ to compensate when this assumption is removed

$$\begin{aligned} \text{revenue}(\text{SBP}) &\leq 4 \text{ revenue}(\text{SBP}) + \text{revenue}(\text{SBP}) \\ &\leq 5 \text{ revenue}(\text{SBP}) \end{aligned}$$

SBP is 5-competitive

- Recall: we assumed all requests served by OPT during a segment are available to SBP
- Need another factor of revenue(SBP) to compensate when this assumption is removed

$$\begin{aligned} \text{revenue(SBP)} &\leq 4 \text{ revenue(SBP)} + \text{revenue(SBP)} \\ &\leq 5 \text{ revenue(SBP)} \end{aligned}$$

And:

$$\text{revenue(SBP)} \geq \text{revenue(OPT)}$$

So:

$$\text{revenue(OPT)} \leq 5 \text{ revenue(SBP)}$$

This bound is tight

Uniform Revenue

- Useful when all requests have equal priority



- SBP is 4-competitive

Online-Dial-a-Ride for Bipartite Graphs

sources



destinations



SBP is $\lceil 1/k \rceil$ -competitive, where k is a fraction used to bound the minimum edge weight in the input graph

Summary of Results + Open Problems

Competitive ratio of SBP for ROLDARP

	Uniform Revenue	Nonuniform Revenue
Weighted General Graphs	4^\dagger	5^\dagger
Weighted Bipartite Graphs	$\lceil 1/k \rceil^*$	$\lceil 1/k \rceil^{*\dagger}$

\dagger with an additive factor

* k is a fraction used to bound the minimum edge weight in the input graph

Thank you!