

Group Steiner problems with degree bounds

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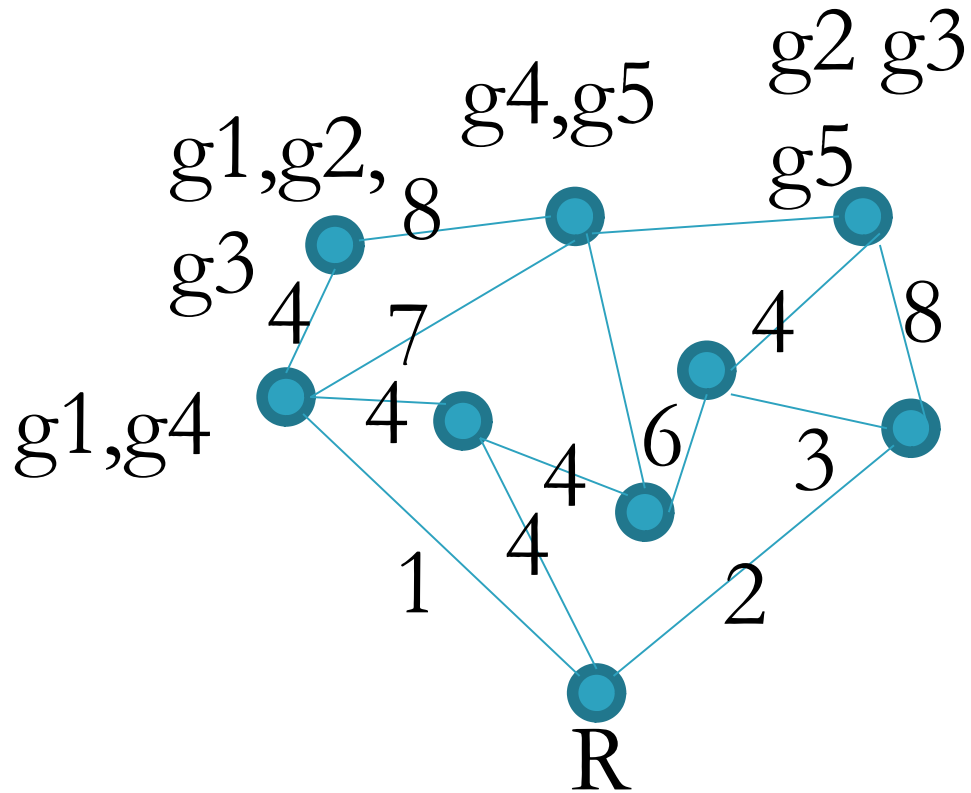
Group Steiner on General Graph

- ▶ Given a graph $G(V,E)$ with edge costs $c(e)$ and a root r .
- ▶ A group is a subset g_i of V .
- ▶ Say: k groups g_1, \dots, g_k
- ▶ Let the maximum size of a group is N .
- ▶ A solution is a Tree $T(V',E')$, rooted at r so that contains at least one vertex of every g_i

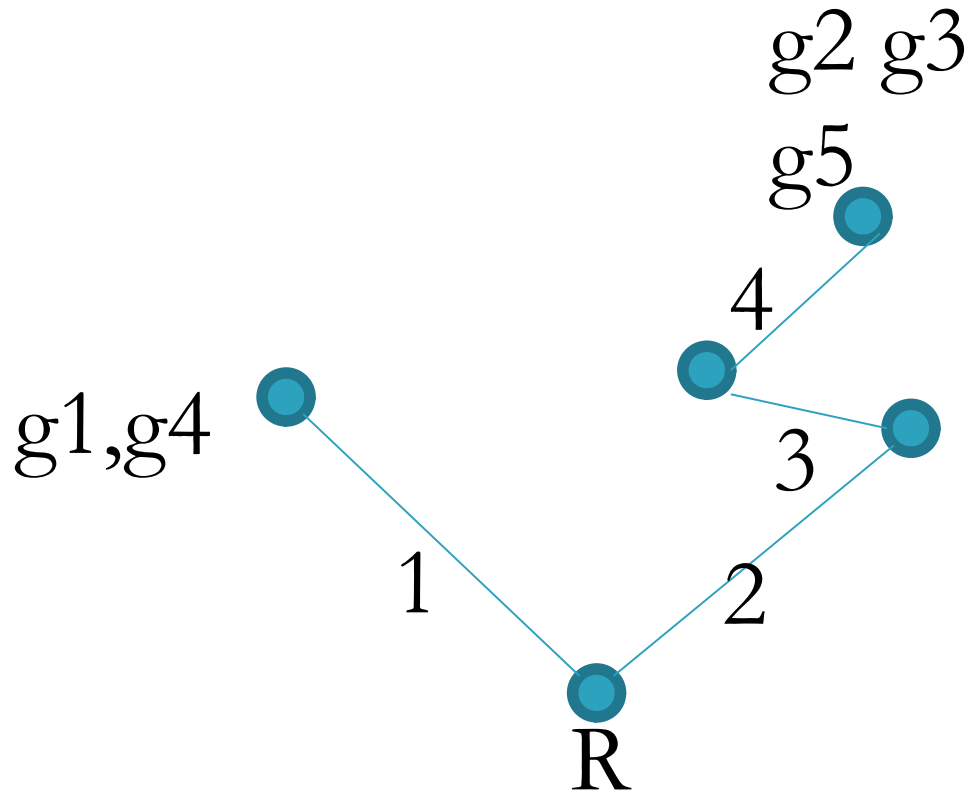
Motivation

- ▶ The motivation came from **VLSI**.
- ▶ Transistors need to be connected by a tree to a central vertex **r**.
- ▶ Every transistor can be located at different ports (the group).
- ▶ Once you choose the port per transistor then you need to choose a tree and connect to **r**.
- ▶ In practice **low degree** are very important for layout and routings. Maybe more important than costs.

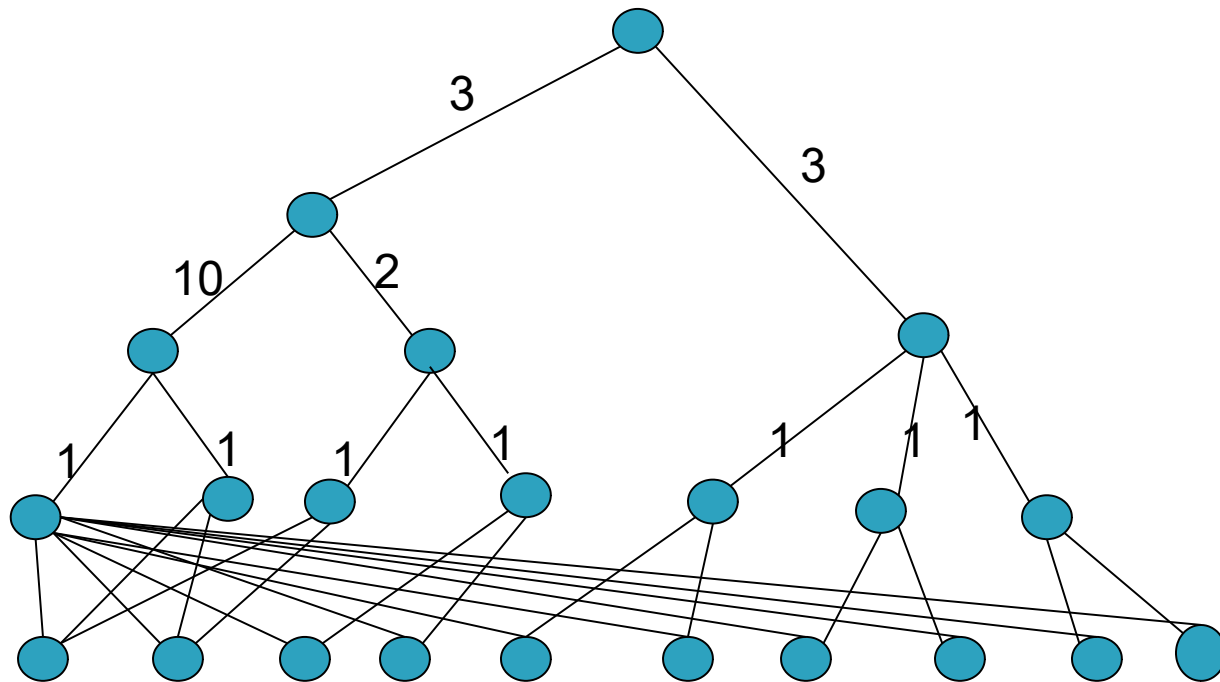
Example



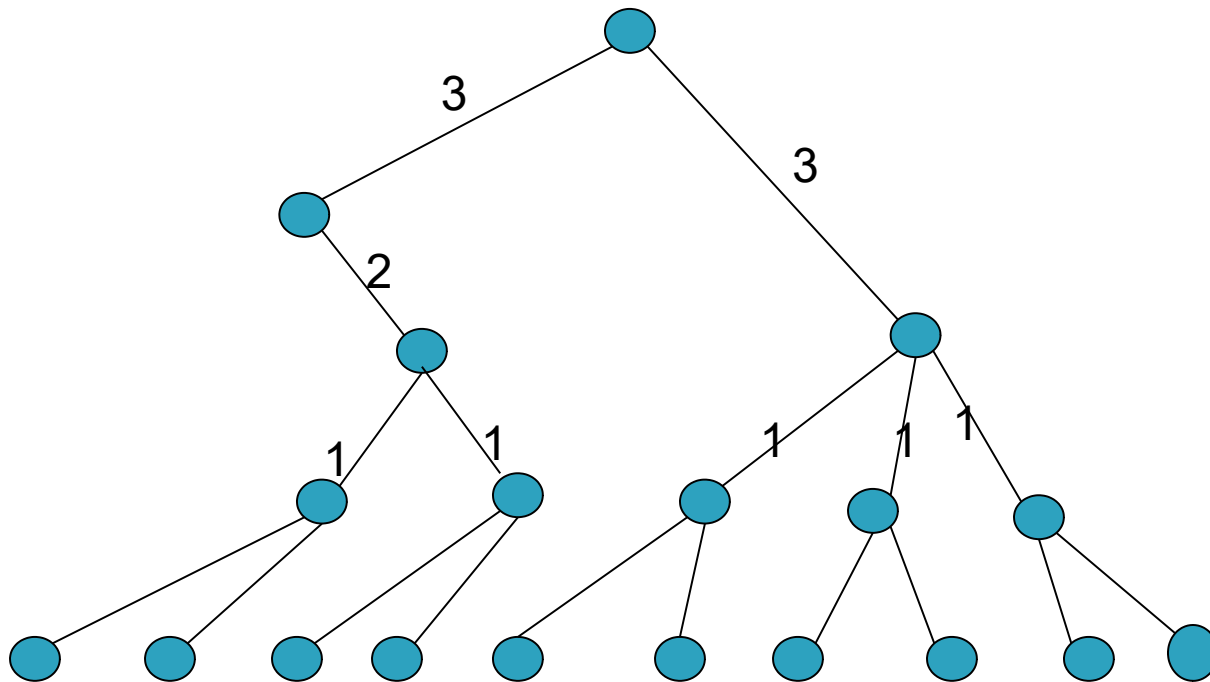
Example



Group Steiner on trees is nothing but Set Cover with tree costs



Choosing a **Set Cover** of size **5** is better than size **1**



Bounding degrees

- ▶ Fakcharoenpho Rao and Talwar reduction does not respect degrees.
- ▶ The reductions of Racke do not work.
- ▶ Only known approximation: For the **Directed Steiner Tree Problem**:
- ▶ $(O(\log^2 n), O(\log^2 n))$ bicriteria in **quasi-poly time**.
- ▶ Due to **Guo, Kortsarz, Laekhanukit, Li, Vaz and Xian**. Manuscript

Our results

- ▶ For trees $(O(\log^2 n), O(\log^2 n))$ and individual degree bounds d_v
- ▶ $(O(\log^2 n) O(\log^3 n))$ was known but not published. A. K. A folklore.
- ▶ $O(\log^3 n)$ ratio for the **Minimum Max Degree Group Steiner** on **Bounded Tree Width** graphs.

Valid inequalities

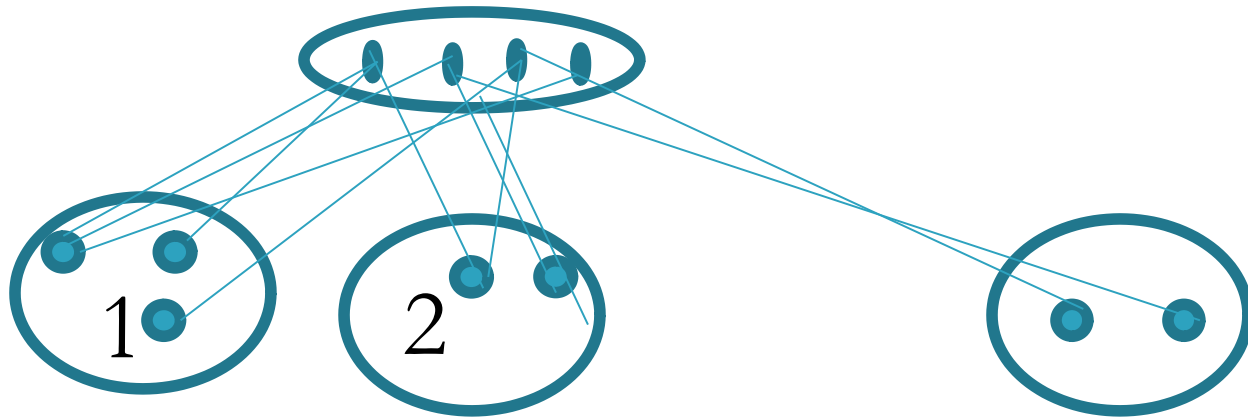
- ▶ Let e_v be the parent edge of v . Let u_1, u_2, \dots, u_p be the children of v in the tree. $e_i = (v, u_i)$
- ▶ $\sum_i x(e_i) \leq x(e_v) * d_v$ is a valid inequality.
- ▶ If $x(e_v) = 0$ then all $x(e_i)$ must be zero for all i .
- ▶ If $x(e_v) = 1$ then $\sum_i x(e_i) \leq d_v$ is a valid inequality.

Polynomial algorithm not even for **BTW** graphs

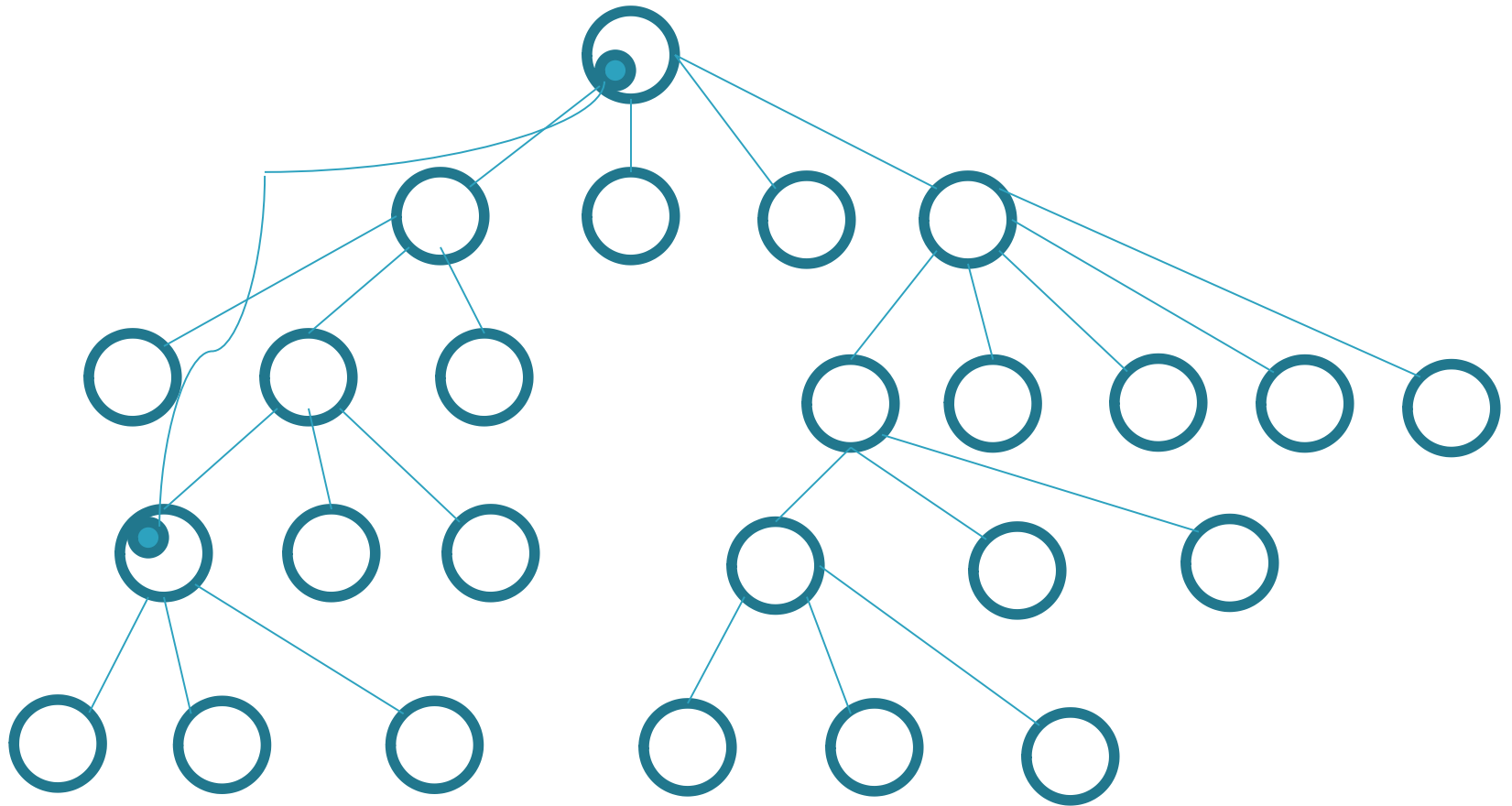
- ▶ However if no degree bounds:
 $O(n^{\text{tw}(G)})$ time $O(\log^2 n)$ ratio for **Minimum Cost Group Steiner** on **BTW** graphs.
- ▶ Due to Chalermsook, Das, Laekhanukit and Vaz
- ▶ Does not use **FRT**.

We deal with no cost case. The goal: minimize the maximum degree on **BTW** graphs.

There is an $O(1)$ separators, CC have size at most $2/3$



A DFS tree of sets of separators



High level $O(\log^3 n)$ for BTW graphs

- ▶ Make sure any parent-child have at least one edge. Select to the separator an extra vertex that has an edge to a vertex in the parent separator.
- ▶ Make all separators connected adding $O(\log n)$ additive factor on degrees.
- ▶ Replace any backward edge in the optimum uses by the path in the tree. Prove it is $O(\log n \cdot d^*)$ additive degree for every vertex.

Paths in the tree

- ▶ We can get from any vertex in a set to any other vertex in another set (separators connected and at least one edge for any parent child).
- ▶ Therefore we contract the separators getting a DFS tree. $O(k)$ multiplication of the degrees in the optimum.
- ▶ Any path in the tree corresponds to a path in the graph.

Connecting separators

- ▶ Let $T(Z)$ be the tree rooted by a separator Z .
- ▶ Fix a vertex u in Z . Add in $T(Z)$ the paths from all other vertices in $Z-u$ to u in $T(Z)$
- ▶ This adds $O(\log n)$ additive ratio.

The way a vertex is affected

- ▶ A vertex is affected from $O(\log n)$ separators, one per every level.
- ▶ This follows from the **DFS** structure of the tree.
- ▶ We add $k+2$ paths that may affect a vertex at every level. The increase of degree is by at most $2(k+2)$ per level.
- ▶ Therefore an additive $O(\log n)$ additive number of edges per vertex. *Its very important we connected downward.*

Once the separators are connected

- ▶ Contract the separators to a vertex. Remove self loops and parallel edges.
- ▶ We got a standard **DFS** tree with **backward edges**.
- ▶ So why did we not start with a **DFS** tree?
- ▶ Because in our tree we have height **$O(\log n)$** .
- ▶ Take **OPT** and replace each backward edge **uv** by the path in the tree between **uv** in the tree.
- ▶ What is the affect on the degrees?

Any vertex is influenced by at most one set per level.

- ▶ In the optimum, if degrees are at most d^* then there will be at most $(k+1) \cdot d^*$ backward edges per separator. Replacing them with paths gives at most $2 \cdot (k+1) \cdot d^*$ additive increase in the degree of many vertices.
- ▶ The total increase is an $O(\log n) \cdot d^*$ additive increase in the degrees for any specific vertex.
- ▶ We transformed the OUTPUT into a tree.

Transforming the input to a tree

- ▶ Since we removed all backward edges, the new solution is a tree. With additive Penalty $O(\log n) \cdot d^*$ increase in the degree.
- ▶ Thus the solution will be a subtree of T' . Namely the tree resulting by removing backward edges.
- ▶ Thus we can change the INPUT to T' and use our algorithm for trees.

Remarks

- ▶ Recently ($O(\log^2 n)$ $O(\log n)$) for minimum degree Group Steiner on trees.
- ▶ Optimal!
- ▶ Gives $O(\log^2 n)$ ratio for Minimum MAX Degree for Group Steiner on BTW graphs.
- ▶ The case of BTW with degree bounds and cost is not known to admit any polylog ratio in polynomial time(!). Open problem.
- ▶ More results omitted due to time limitation.